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ABSTRACT

This framework document describes the content and format of the National Assessment of Educational Progress (NAEP) mathematics assessments of 1996, 2000, and 2003. Five content strands are discussed in the NAEP mathematics assessment: (1) number sense, properties, and operations; (2) measurement; (3) geometry and spatial sense; (4) data analysis, statistics, and probability; and (5) algebra and functions. The level of mathematical ability, including conceptual understanding, procedural knowledge, and problem solving, is regarded as playing a central role in defining item descriptors and achieving balance across the tasks for each grade level in the NAEP mathematics assessment. The framework reflects an integrated view of school mathematics. Percentage of items allotted to each of the five strands, families of tasks/items to measure the depth of student knowledge in mathematics, items requiring students to construct a response, manipulative materials used to measure student knowledge and problem-solving abilities, and review for potential item bias are also discussed. (KHR)

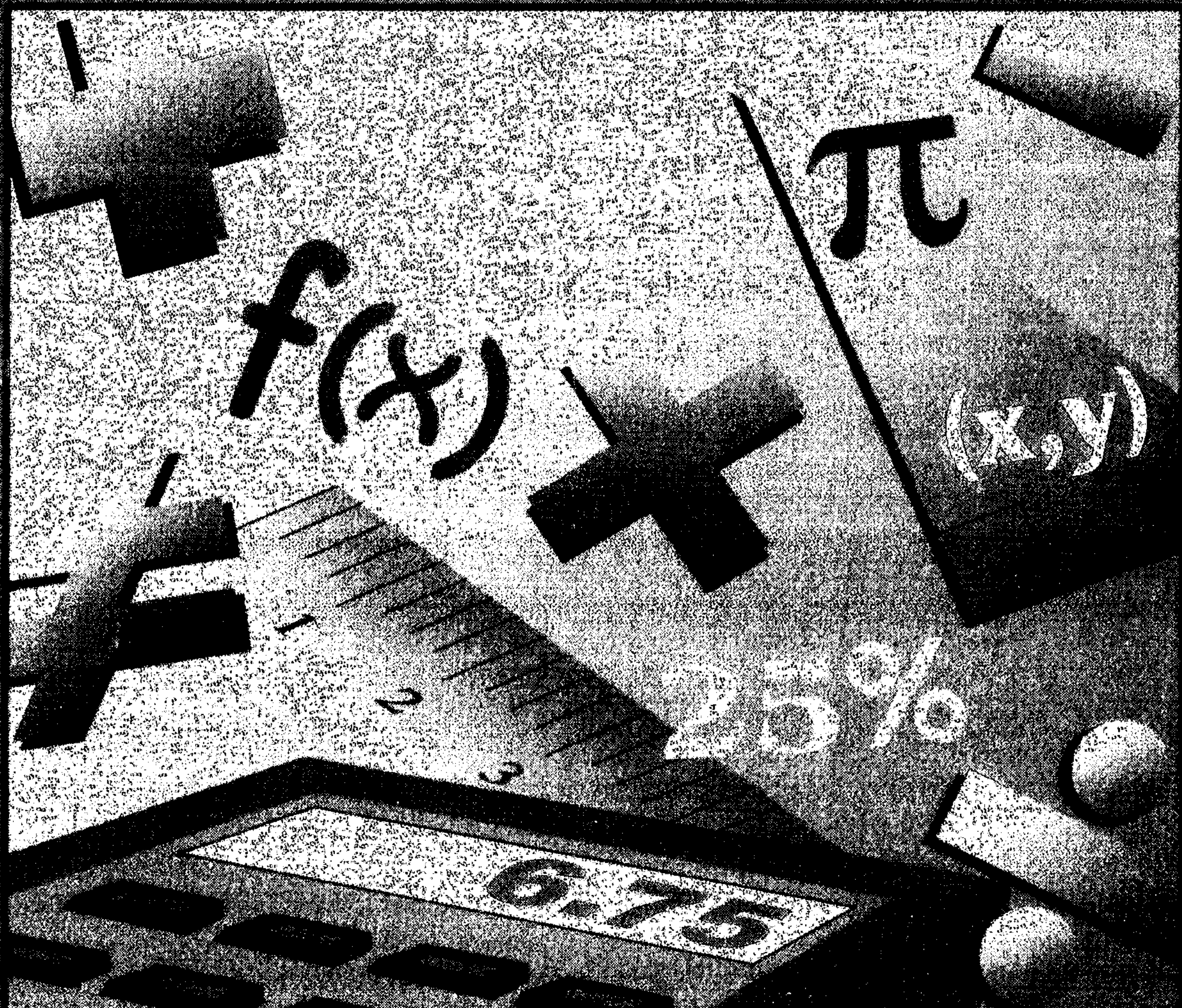
ED 470 533

# Mathematics Framework for the 2003 National Assessment of Educational Progress

## National Assessment Governing Board

### U.S. Department of Education

# Mathematics Framework for the 2003 National Assessment of Educational Progress




National Assessment Governing Board

U.S. Department of Education

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# What Is NAEP?

The National Assessment of Educational Progress (NAEP) is the only nationally representative and continuing assessment of what America's students know and can do. It is a congressionally mandated project of the U.S. Department of Education's National Center for Education Statistics. NAEP measures student achievement in reading, mathematics, writing, science, U.S. history, geography, civics, the arts, and other subjects. Since 1969, NAEP has surveyed the achievement of students at ages 9, 13, and 17 and, since the 1980s, in grades 4, 8, and 12.

## The National Assessment Governing Board

The National Assessment Governing Board (NAGB) was created by Congress to formulate policy for NAEP. Among the Board's responsibilities are developing objectives and test specifications and designing the assessment methodology for NAEP.

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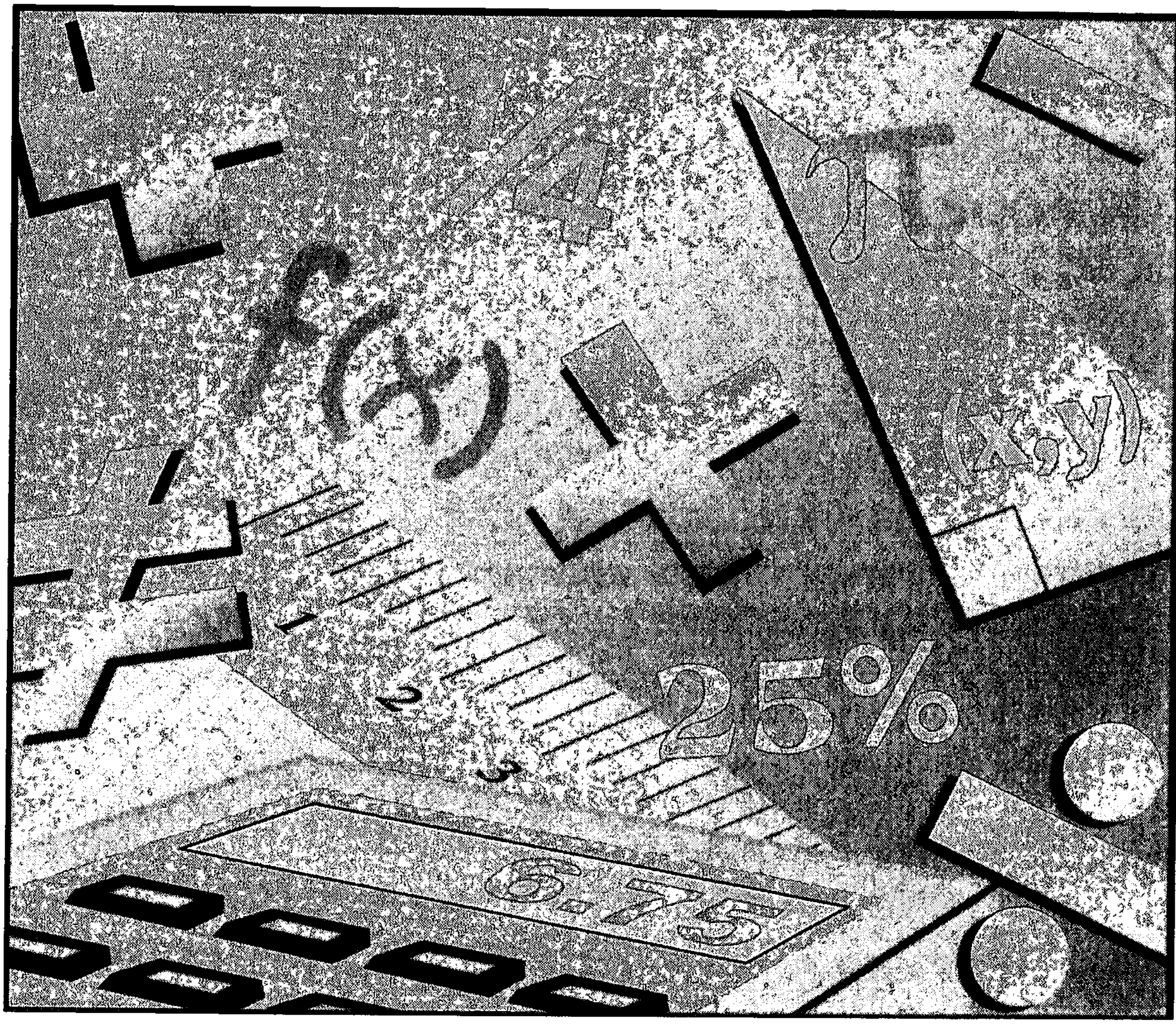
# Mathematics Framework for the 2003 National Assessment of Educational Progress

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## Mathematics Framework for the 2003 National Assessment of Educational Progress

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# Contents

# Chapter One

## Overview

### Introduction

Since 1973, the National Assessment of Educational Progress (NAEP) has gathered information about levels of student proficiency in mathematics and the related practices of teachers in our nation's schools. The results of these periodic assessments are published in *The Nation's Report Card* to inform citizens about the nature of students' comprehension of the subject, curriculum specialists about the level and nature of student understanding, and policymakers about factors related to schooling and its relationship to student proficiency in mathematics.

Based on these surveys of students at the end of elementary, middle, and high school, *The Nation's Report Card* has provided comprehensive information about what students in the United States know and can do in the area of mathematics and in several other subjects. These reports present information about strengths and weaknesses in students' understanding and their ability to apply that understanding in problem-solving situations; provide comparative student data according to race/ethnicity, type of community, and geographic region; describe trends in student performance over time; and report relationships between student proficiency and certain background variables. This framework document describes the content and format of the NAEP mathematics assessments in 1996, 2000, and 2003. Although there have been revisions to the framework, NAEP has maintained the mathematics trend begun in 1990.

### Context for Planning the Mathematics Assessment

The National Assessment Governing Board (NAGB), created by Congress in 1988, is responsible for formulating policy for NAEP. NAGB is specifically charged with developing assessment objectives and test specifications, identifying appropriate achievement levels, and carrying out other NAEP policy responsibilities. In 1990,

the U.S. Department of Education conducted the first voluntary state-by-state assessment of mathematics as an adjunct to its periodic NAEP national assessments of mathematics. The 1990 state-level trial was limited to the 8th grade. In 1992, the second voluntary state-level assessments associated with NAEP were carried out at the fourth- and eighth-grade levels in mathematics and at the fourth-grade level in reading. Current NAEP legislation in the No Child Left Behind Act of 2001 requires that NAEP assess mathematics and reading every two years at the national and state levels in grades 4 and 8. This schedule begins with the 2003 assessment.

To prepare for the 1990 trial state assessment, the National Center for Education Statistics awarded a contract in 1987 to the Council of Chief State School Officers (CCSSO) to design a framework for the assessment. The Mathematics Framework Project gave special attention to the nature of formal state objectives and frameworks for mathematics instruction. In doing so, the Framework panels sampled state-, district-, and school-level objectives; examined the curricular frameworks on which previous NAEP assessments were based; consulted with leaders in mathematics education; and reviewed a draft version of the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics*. This project resulted in the "content-by-mathematical-ability" matrix design used to guide both the 1990 and 1992 NAEP mathematics assessments.

To prepare for the next NAEP mathematics assessment, NAGB awarded a contract in fall 1991 to The College Board to develop assessment and item specifications for the 1994 mathematics assessment.

The process of developing the recommendations for the planned 1994 NAEP mathematics assessment occurred between September 1991 and March 1992. Because of a budget shortfall, however, both the new NAEP mathematics and science assessments were rescheduled from 1994 to 1996.

The NAEP mathematics project conducted by The College Board had two primary purposes. The first was to recommend a framework for the overall design of the mathematics assessment—that is, a structure for describing what students should know and be able to

do in mathematics. The second was to develop specifications for the assessment items, with particular attention to a mix of formats, the item distribution for content areas within mathematics, and the conditions under which items are presented to students (e.g., use of manipulatives, use of calculators, and other factors).

The new *NAEP Mathematics Framework* was considered in light of the three NAEP achievement levels—Basic, Proficient, and Advanced.

## Achievement Levels

**Basic** denotes partial mastery of prerequisite knowledge and skills that are fundamental for proficient work at each grade.

**Proficient** represents solid academic performance for each grade assessed. Students reaching this level have demonstrated competency over challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter.

**Advanced** represents superior performance.

These levels are intended to provide descriptions of what students *should* know and be able to do in mathematics. Established for the 1992 mathematics scale through a broadly inclusive process and adopted by NAGB, the three levels per grade are the primary means of reporting NAEP data. The new mathematics assessment was constructed with these levels in mind to ensure congruence between the levels and the test content. See appendix A for the NAEP Mathematics Achievement Level Descriptions.

## Framework and Specifications Development Process

The College Board convened a steering committee representing national education organizations, policymakers, and business to review the direction and scope of the project. A planning committee of mathematics educators met to draft the assessment framework. Both committees considered (1) the status of national reform efforts in mathematics education and assessment evaluations of the NAEP trial state assessment in mathematics (Silver, Kenney, and Salmon-Cox, 1991) and (2) the fit between NAEP assessments and

the teaching of mathematics at grades 4, 8, and 12 in the nation's schools (Romberg, Wilson, Smith, and Smith, 1991). Committee members are listed in appendix B.

The suggested revisions in the framework are intended to reflect the 1990 and 1992 NAEP assessments. Committee members also made use of the findings of evaluation studies concerning the NAEP assessments. The findings of these studies were merged with research and state standards for the assessment of student proficiency in mathematics. Finally, the committee reviewed information provided by the 1990 assessment, noting features of the framework and how those features assisted or hindered the clear understanding of what students knew and were able to do in mathematics appropriate to their ages and levels of education. Another important phase in the process involved conducting a national mail review and convening focus groups in six states to gather input on the committee's recommendations.

The suggested revisions in the framework for the new NAEP assessment in mathematics are intended to reflect curricular emphases and objectives; include what various scholars, practitioners, and interested citizens believe should be in the assessment; and maintain ties to previous assessments to permit the reporting of trends in student achievement across time.

## Recommendations for the 1996 and Future NAEP Mathematics Assessments

As a result of analysis and review, the steering committee and planning committee endorsed the following recommendations for the 1996 and future NAEP mathematics assessments:

1. **Content Strands**

   The matrix framework employed in the 1990 and 1992 NAEP assessments should be discontinued in favor of a model consisting primarily of the five major content strands used in that matrix model. Evaluation studies of the NAEP trial state assessment and other cognitive science recommendations dealing with assessment suggest that forcing content into a rigidly structured, content-by-ability-level matrix distorts the nature

of the discipline. A model that calls for the assessment of knowledge in discrete, content-by-ability-level categories is inappropriate in an era in which more progressive recommendations call for attention to a student's ability to connect knowledge in one area of mathematics with knowledge and abilities in other areas of mathematics.

Therefore, the recommendation was to use the five major content strands: (1) Number Sense, Properties, and Operations; (2) Measurement; (3) Geometry and Spatial Sense; (4) Data Analysis, Statistics, and Probability; and (5) Algebra and Functions. These strands have their foundation in NAEP mathematics assessments beginning in the 1970s. The nature of the strands is further discussed in chapters two and three.

**2. Mathematical Abilities**

The levels of mathematical ability (conceptual understanding, procedural knowledge, and problem solving) should not be used to define specific percentages of items in each of the five content strands, as was done in the 1990 and 1992 assessments. However, these descriptors, along with the more encompassing process goals of reasoning, connections, and communication, should play a central role in defining item descriptors and achieving balance across the tasks for each grade level in the NAEP mathematics assessment. This recommendation is discussed further in chapters two and four.

**3. Percentage of Items**

The percentage of items allotted to each of the five strands should continue the move begun with the 1990 assessment toward a balance among the five strands and away from an assessment dominated by number and operations. The recommendations, although retaining a core of items that reflect traditional goals in the Basic skills, represent continued movement toward a broad algebra- and geometry-oriented program at the eighth- and twelfth-grade levels. The specific percentage of items recommended is further discussed in chapter two.

## 4. Item Families

To measure the breadth and depth of student knowledge in mathematics, “families” of tasks/items should be created for each grade level of the assessment. A family of tasks/items is a related set of assessment tasks that can probe the vertical or horizontal nature of a student’s understanding. A vertical family might include items that measure students’ abilities to define a concept, apply the concept in a familiar setting, use the concept or related principles to solve a new problem, and ultimately generalize knowledge about the concept or related principles to represent a new level of understanding. A vertical family might lie within a single grade level or extend across grade levels. Another family of items might measure students’ horizontal understanding of a concept or principle across content strands. For example, students’ proficiency in solving the proportion 2/3 = 16/x might be measured in a number context, in a measurement setting, in a geometry setting, in a probability setting, and in an algebraic setting. Students’ ability to work with the proportion in each of these contexts tells a great deal about the richness of their understanding of the concept and the related procedural skills.

## 5. Constructed-Response Items

The number of items requiring students to construct a response should be increased as much as possible within the bounds of the statistical design used to carry out the assessment. Furthermore, these items provide excellent opportunities to measure students’ abilities to reason mathematically as well as connect and communicate their knowledge of mathematics. In particular, the number of extended open-ended items should be increased from the number given in the 1992 assessment.

## 6. Special Studies

At the twelfth-grade level, a special study should be carried out using graphing calculators to establish baseline data for gradually introducing these calculators, which can assist students in visualizing algebraic relations, into the curriculum.

## 7. Manipulatives

The assessment should continue using reasonable manipulative materials, where possible, to measure students' knowledge and problem-solving abilities. Such manipulative materials and accompanying tasks should be carefully chosen to minimize disruption of the test administration process.

## 8. Item Bias Review

Although bias analysis is consistently conducted on NAEP items and student performance as mandated by law, recommendations for shifting the types of items used on the assessment merit an especially careful look at potential item bias. Data should be gathered during field testing and during the actual assessment and analyzed for any unforeseen item bias that may arise from incorporating less widely used types of assessment items. The 1996 and future NAEP assessments will incorporate awareness of this critical consideration, especially related to students' previous opportunities to learn and their experience and background both in school and outside of school. Sensitivity and a sound research base will guide not only test construction but also the reporting of student performance.

These recommendations were made in an attempt to reflect the increasing realization that student proficiency in mathematics is not the result of the interaction of discrete cells of knowledge with a discrete list of special mathematical abilities. Rather, student proficiency in mathematics results from broad experience in forming networks of connections among mathematical ideas and skills. The current framework and specifications reflect a more integrated view of school mathematics than previous NAEP frameworks.

**Note:** For the 2005 mathematics assessment, the Governing Board conducted a comprehensive Framework Update Project to further enhance the current framework and specifications. Although changes were recommended at grades 4 and 8, these modifications are not expected to disrupt the trends begun in 1990. More substantial changes were recommended for the 12th-grade assessment. Refer to the NAGB Web site at *www.nagb.org* for further information on the *2005 Mathematics Framework.*

# Chapter Two

# Framework for the Assessment

## Content Strands

This chapter further discusses the rationale for recommendations presented in chapter one. The framework for the NAEP mathematics assessment is anchored in broad strands of mathematical content:

- Number Sense, Properties, and Operations
- Measurement
- Geometry and Spatial Sense
- Data Analysis, Statistics, and Probability
- Algebra and Functions

These strands are not intended to divide mathematics into discrete elements. Rather, they are intended to provide a helpful classification scheme that describes the full spectrum of mathematical content assessed by NAEP.

## Mathematical Dimensions

The 1990 and 1992 NAEP mathematics assessments made use of matrix frameworks to specify items by both content strand and mathematical ability, as shown in figure 1. The use of such frameworks provided strong guidance for the construction of the assessment in terms of breadth. Nonetheless, this type of structure tended to work against the curricular goal of integrating mathematical knowledge across topics.

**Figure 1. Framework for the 1990 and 1992 Mathematics Assessments**

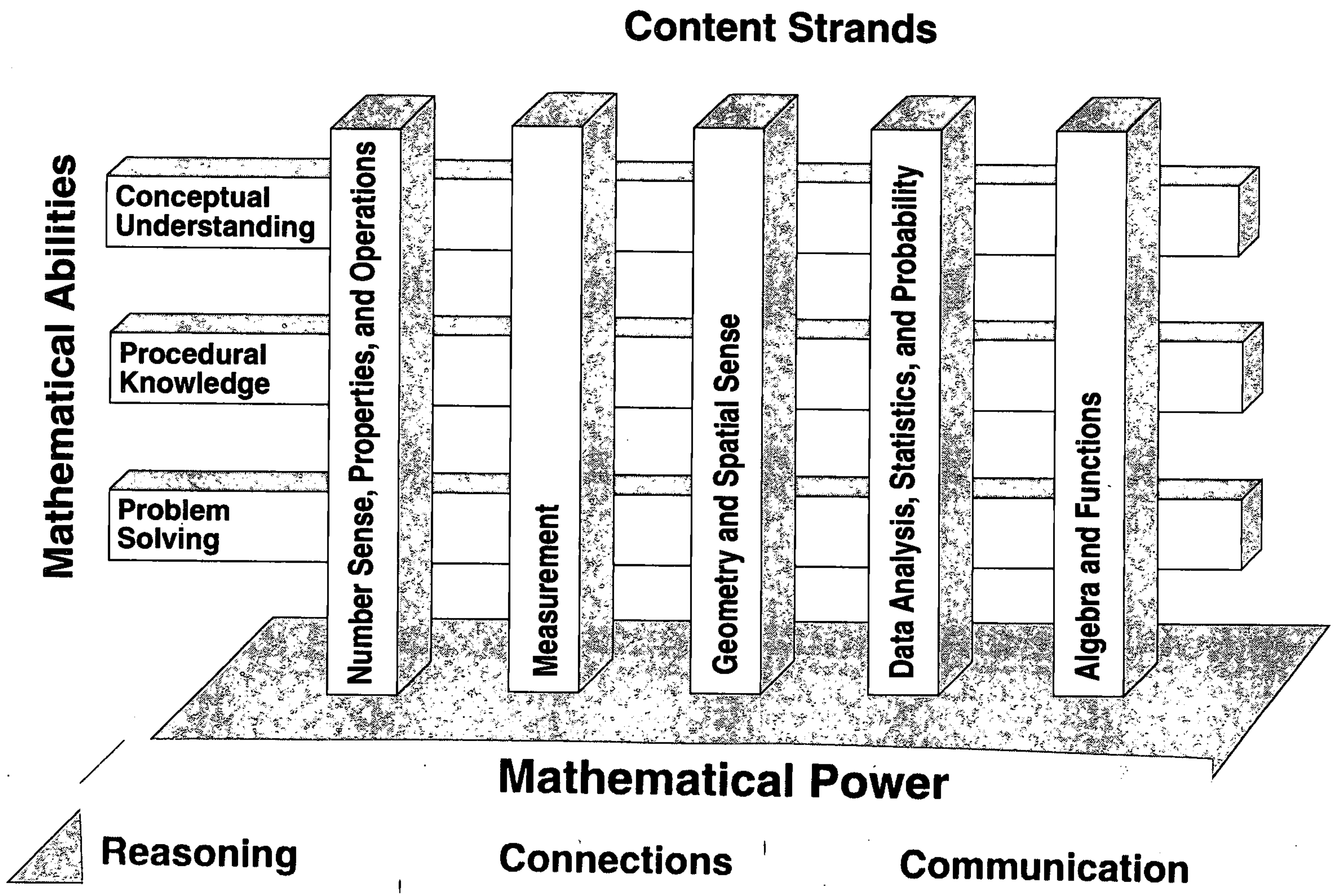
| Mathematical Abilities | Content Areas | | | | |
|---|---|---|---|---|---|
| | Numbers and Operations | Measurement | Geometry | Data Analysis, Statistics, and Probability | Algebra and Functions |
| Conceptual Understanding | | | | | |
| Procedural Knowledge | | | | | |
| Problem Solving | | | | | |

Additionally, on secondary analyses of the NAEP items, expert panels often had difficulty replicating the assignment of items to cells of the matrix on the basis of the mathematical ability classifications. Classifications varied with the rater's conceptions of students' abilities of children in grades 4, 8, or 12 rather than with the definitions of the mathematical abilities. The strict application of the mathematical abilities classifications in conjunction with the content strands led to a forced fit of items to achieve balance across the two-dimensional matrix rather than to match the goals of mathematics education.

In real life, few mathematical situations fall clearly in one content strand or another, and few naturally reflect only one facet of mathematical thinking. Yet, to ensure a broad scope in test construction, items must be classified several ways. To address this issue of item classification, the current framework for mathematics assessments focuses primarily on the mathematical content strands, with additional specifications related to an assessment dimension referred to as "mathematical power," as shown in figure 2.

Figure 2 shows that the curriculum is conceived as consisting of content drawn from five broad mathematical areas. Items are classified according to the major area(s) they address, including both mathematical abilities and mathematical power. Mathematical

Figure 2. Framework for the 1996, 2000, and 2003 Mathematics Assessments



power is conceived as consisting of mathematical abilities (conceptual understanding, procedural knowledge, and problem solving) within a broader context of reasoning and with connections across the scope of mathematical content and thinking. Communication is viewed as both a unifying thread and a way for students to provide meaningful responses to tasks.

In recent NAEP administrations, the concept of mathematical power as reasoning, connections, and communication played an increasingly important role in measuring student achievement. In 1990, the assessment included short-answer open-ended items as a way to begin to address mathematical communication. The extended open-ended items included on the 1992 assessment required students not only to communicate their ideas but also to begin to demonstrate the reasoning they used to solve problems. The new assessment items focus even more attention on mathematical power by continuing deliberate attention to reasoning and communication and by providing students with opportunities to connect their learning across mathematical content strands. These connections are addressed through individual items designed to tap more than one content strand or more than one ability, as well as across items through the use of item families.

## Families of Items

Families of related items were designed to sample the depth of students' knowledge within a particular strand and students' ability to deal with concepts, principles, or procedures across content strands. Within a family, items may cross content areas, mathematical abilities, and/or grade levels. This type of grouping in the design of the assessment allows a more indepth analysis of student performance than would a collection of discrete items. Individual student performance, comparisons of student performance across grade levels and strands, and comparisons of student performance across assessments with respect to a family of items provide another way to assess areas of strength and weakness.

A more detailed discussion of each content strand is provided in chapter three, and more detailed descriptions of item types are provided in chapter five.

## Percentage of Items

The distribution of items among the various mathematical content strands is a critical feature of the assessment design, as it reflects the relative importance and value given to each strand. In the past six NAEP assessments in mathematics, the categories received differential emphasis, and the differentiation continues in the framework for the 1996, 2000, and 2003 assessments. The recommended distribution of items to the strands continues to move toward a more even balance among the strands and away from the earlier model, in which items reflecting number facts and operations composed more than 50 percent of the assessment item bank.

Another significant difference in the new assessment is that items may be classified in more than one strand. In addition to describing minimum percentages of the item pool that should address each strand, note that maximum percentages are listed for the Number Sense, Properties, and Operations strand to ensure that the balance is maintained. Table 1 provides the recommended mix of items in the assessment by content strand for each grade (4, 8, and 12).

These guidelines for balance present a minimum target for representation across mathematical content strands. For Number Sense, Properties, and Operations, note that a maximum target is also provided. This target is intended to reinforce the shift away from a narrow number and computation focus to a more comprehensive view of mathematics. An item should be classified according to its predominant strand; it may be classified under two or more strands if it addresses substantive content from more than one area. In fact, at least half of the new items should have major elements drawn from more than one strand, and they should be categorized in those strands. This means that the percentages listed in table 1, when translated into data on the actual item pool, will result in a percentage of items greater than those listed and will add up to more than 100 percent. Additionally, the number of items reflecting connections among strands should continue to increase in subsequent assessments to move NAEP assessments ever closer to the

## Table 1. Minimum Percentage Distribution of Items, by Grade and Content Strand

| Content Strand | Grade 4 | Grade 8 | Grade 12 |
|---|---|---|---|
| Number Sense, Properties, and Operations* (minimum/maximum) | 40/70 | 25/60 | 20/50 |
| Measurement | 20 | 15 | 15 |
| Geometry and Spatial Sense | 15 | 20 | 20† |
| Data Analysis, Statistics, and Probability | 10 | 15 | 20 |
| Algebra and Functions | 15 | 25 | 25 |

Note: An item may be classified in more than one category.

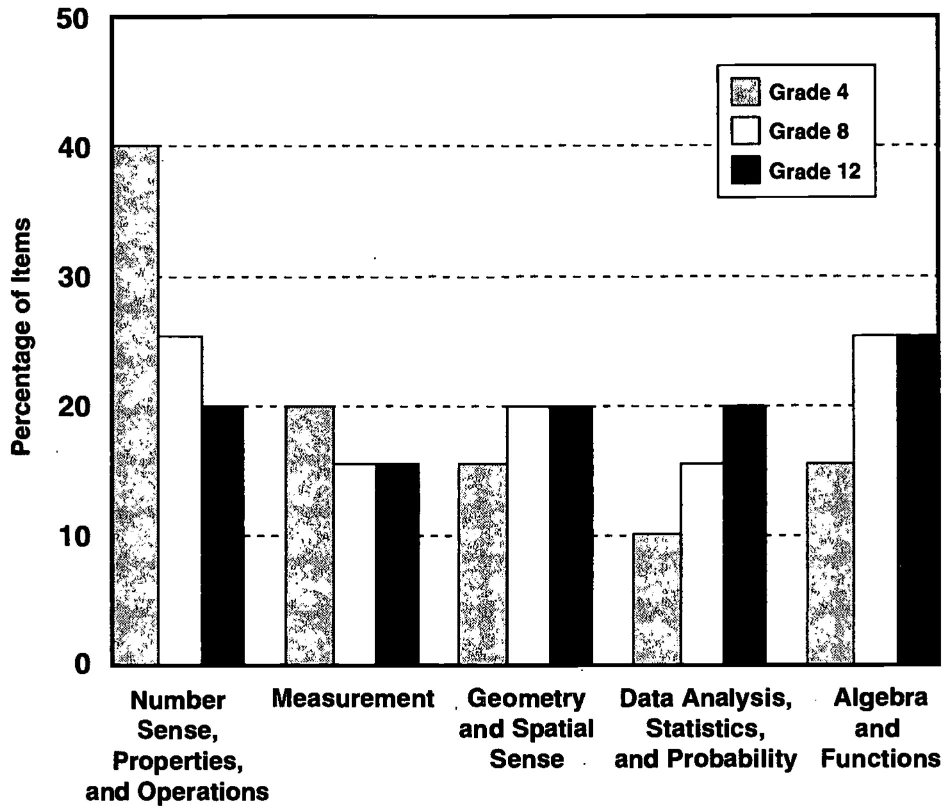
*At least half of the items in Number Sense, Properties, and Operations at each grade level should involve some aspect of estimation or mental mathematics. No more than the specified maximum percent of the items at any grade level should have a major classification in this strand.

†At grade 12, 25 percent of the items in the geometry strand should involve topics in coordinate geometry.

goal of students having the opportunity to demonstrate mathematical power in various situations requiring connections within mathematics and with other disciplines.

Figure 3 shows the percentage of items by content strand and grade level on the NAEP mathematics assessment. The emphasis given to Number Sense, Properties, and Operations in grade 4 shifted toward growing emphases in Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions in the later grades.

**Figure 3. Percentage of Items, by Content Strand and Grade Level (Minimum percentages shown)**




## Item Balance

Mathematical power can be thought of as an extension of "mathematical abilities," as the term was used in the 1990 and 1992 mathematics assessments. The mathematical abilities described in the framework for these assessments (procedural knowledge, conceptual understanding, and problem solving) specifically addressed aspects of knowing and doing mathematics. Nonetheless, the development of assessment items based only on a rigid content-by-process matrix has led to a contrived separation and artificial contexts. Indeed, expert reviewers of the 1990 assessment often were unable to agree on the best placement for some items in the framework matrix.

The current specifications are designed to incorporate the overarching standards for communicating, reasoning, and connecting, as well as the categories of conceptual understanding, procedural knowledge, and problem solving. The following recommendations are intended to guide the development of actual items for the 1996 and future NAEP mathematics assessments. These guidelines are

provided to assist in reviewing the overall balance in the assessment and ensure that the assessment reflects some balance among "knowing that or knowing about," "knowing how," and "solving problems," within an overall demonstration of mathematical thinking in various situations. Chapter four includes a more indepth discussion of mathematical power, mathematical abilities, and additional aspects of mathematical thinking as they relate specifically to the current and future mathematics assessments.

Guidelines for balancing the mix of conceptual understanding, procedural knowledge, and problem-solving items should be evaluated only in terms of the total item package at each grade level, not in each individual strand. As in the content classification, classification according to these three mathematical abilities need not—in fact should not—be forced into individual categories. Rather, an item will likely include elements of more than one of these three, and it should be classified in as many of these categories as is appropriate for the major thought processes required.

At each grade level, at least one-third of the items should be classified as conceptual understanding, at least one-third should be classified as procedural knowledge, and at least one-third should be classified as problem solving. Items with a major element of procedural knowledge in addition to either conceptual understanding or problem solving should not make up the majority of items at any grade level.

To present a more complete picture of national mathematics performance, there should be an increase in the total number of items in the assessment and the number of items requiring student-constructed responses. In particular, any increase should reflect at least a doubling of the number of extended open-ended items contained in the 1992 NAEP assessment and an attempt to equalize the number of short-answer and multiple-choice items.

The percentage distributions presented here, the lists of topics provided in chapter three, and the described elements of mathematical power are not intended to prescribe curriculum standards; rather, they are designed to construct a complete and balanced assessment

instrument reflecting best practice in mathematics education at each grade level. An analysis of student performance across all items will permit NAEP to report on average mathematics proficiency. In addition, analysis of performance on subsets of items will permit reporting on patterns of achievement in each of the five strands.

## Calculators

In recent NAEP assessments, students have been provided calculators to gather information on certain blocks of items measuring ability to use calculators in mathematical situations. However, some items require students to demonstrate computation or estimation skills without the use of a calculator.

In the 1996, 2000, and 2003 assessments, calculators were provided on about one-third of the assessment. Students do not have access to a calculator on the about two-thirds of the exam. At grade 4, NAEP provides students with a four-function calculator. At grades 8 and 12, students are provided with a scientific calculator by NAEP.

## Manipulatives

Starting with the 1990 assessment, students were provided rulers and protractors for use on some tasks on the assessments. With the 1992 assessment, students received some geometric shapes to use in responding to items requiring the analysis of relationships between these shapes and more complex shapes that could be formed from the pieces. Assessments in 1996, 2000, and 2003 expanded this practice, especially in settings in which students are given extended time to work with materials that can be easily included in such a large-scale assessment.

# Chapter Three

## NAEP Mathematics Objectives: Content Areas and Assessment Strands

To conduct a meaningful assessment of mathematics proficiency, it is necessary to measure students' proficiencies in various content strands. As in the 1990 and 1992 assessments, five content strands will be used to categorize content for the mathematics assessments. The strands are illustrated later in this chapter. Classification of topics into these strands cannot be exact and inevitably will involve some overlap. For example, some topics appearing under Data Analysis, Statistics, and Probability may be closely related to others that appear under Algebra and Functions. As assessment programs continue to be refined, it becomes less desirable to force every item into only one content strand. Students are expected to solve problems that naturally involve more than one specific mathematical topic. Consequently, the assessment as a whole will address the topics and subtopics identified in this chapter, and every item will be categorized under primarily one topic and subtopic so that analysis of results may be somewhat specific. Ideally, however, the items will require students to synthesize knowledge across topics and subtopics, and occasionally it may be difficult to identify a unique topic for each item. In fact, at least half of the new items for the assessment should involve content from more than one topic or even from more than one strand.

The following sections of this chapter provide a brief description of each content strand with a list of topics and subtopics to be included in the assessment. This level of specificity is needed to guide item writers and ensure adequate coverage of the content areas and abilities to be assessed. The five content strands for 1996, 2000, and 2003 are largely consistent with the strands used in the 1990 and 1992 assessments.

For each grade (4, 8, and 12), the following symbols are used: a "○" indicates that the subtopic can be assessed at that grade level, a

"△" indicates that the subtopic should not be assessed at that grade level, and a "#" indicates that the subtopic may be introduced at a simple level, probably using a manipulative or pictorial model. The test specifications include additional detail and descriptions of how item types, families, calculators, manipulatives, and special studies fit within and across topics and subtopics.

## Number Sense, Properties, and Operations

This strand focuses on students' understanding of numbers (whole numbers, fractions, decimals, integers, real numbers, and complex numbers), operations, and estimation, and their application to real-world situations. Students are expected to demonstrate an understanding of numerical relationships expressed in ratios, proportions, and percentages. Students are also expected to understand properties of numbers and operations, generalize from numerical patterns, and verify results.

Number sense includes items that address a student's understanding of relative size, equivalent forms of numbers, and his or her use of numbers to represent attributes of real-world objects and quantities. Items that call for students to complete open sentences involving basic number facts are considered part of this content strand. Items that require some application of the definition of operations and related procedures are classified under Algebra and Functions.

The emphasis in computation is on understanding when to use an operation, knowing what the operation means, and being able to estimate and use mental techniques in addition to performing calculations using computational algorithms. In terms of actual computation, students are expected to demonstrate that they know how to perform basic algorithms and, in more complex situations, use calculators appropriately. Although some isolated computation items are included, a priority is placed on developing items in which mathematical operations are used in problem-solving situations.

The grade 4 assessment emphasizes the development of number sense through the connection of various models to their numerical representations, as well as an understanding of the meaning of addition, subtraction, multiplication, and division. These concepts are

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addressed for whole numbers, simple fractions, and decimals at this grade level, emphasizing the use of models and their connection to the use of symbols.

The grade 8 assessment extends number sense to include both positive and negative numbers and addresses properties and operations involving whole numbers, fractions, decimals, integers, and rational numbers. The use of ratios and proportional thinking to represent situations involving quantity is a major focus at this grade level, and students are expected to read, use, and apply scientific notation to represent large and small numbers.

At grade 12, the assessment includes both real and complex numbers and tests students' competency in topics up to and including precalculus. Operations with powers and roots, as well as various real and complex numbers, may be assessed. Including a broad range of items at this level ensures that students who have had different types of high school mathematics courses will be able to demonstrate proficiency in some parts of this content area.

**NAEP Mathematics Content Strand 1**

| | Grade | | |
|---|---|---|---|
| **Number Sense, Properties, and Operations** | **4** | **8** | **12** |
| 1. Relate counting, grouping, and place value | | | |
| a. Use place value to model and describe whole numbers and decimals | ● | ● | ● |
| b. Use scientific notation in meaningful contexts | △ | ● | ● |
| 2. Represent numbers and operations in a variety of equivalent forms using models, diagrams, and symbols | | | |
| a. Model numbers using set models such as counters | ● | △ | △ |
| b. Model numbers using number lines | ● | ● | △ |
| c. Use two- and three-dimensional region models to describe numbers | ● | ● | ● |

| | |
|---|---|
| ● | Subtopic can be assessed at this grade level. |
| △ | Subtopic should not be assessed at this grade level. |
| # | Subtopic may be introduced at a simple level (such as using a manipulative or pictorial model). |

| Number Sense, Properties, and Operations | Grade 4 | Grade 8 | Grade 12 |
|---|---|---|---|
| d. Use other models appropriate to a given situation (for example, draw diagrams to represent a number or an operation; write a number sentence to fit a situation or describe a situation to fit a number sentence; interpret calculator or computer displays) | ● | ● | ● |
| e. Read, write, rename, order, and compare numbers | ● | ● | ● |
| 3. Compute with numbers (that is, add, subtract, multiply, divide) | ● | ● | ● |
| a. Apply basic properties of operations | ● | ● | ● |
| b. Describe effect of operations on size and order of numbers | ● | ● | ● |
| c. Describe features of algorithms (such as regrouping with or without manipulatives, partial products) | ● | ● | ● |
| d. Select appropriate computation method (such as pencil and paper, calculator, mental arithmetic) | ● | ● | ● |
| 4. Use computation and estimation in applications | ● | ● | ● |
| a. Round whole numbers, decimals, and fractions in meaningful contexts | ● | ● | ● |
| b. Make estimates appropriate to a given situation | | | |
| i. Know when to estimate | ● | ● | ● |
| ii. Select appropriate type of estimate (overestimate, underestimate, range of estimate) | ● | ● | ● |
| iii. Describe order of magnitude (estimation related to place value, scientific notation) | ● | ● | ● |
| c. Select appropriate method of estimation (such as front end, rounding) | ● | ● | ● |
| d. Solve application problems involving answers or estimates as appropriate | ● | ● | ● |
| e. Interpret round-off errors using calculators/computers (that is, truncating) | △ | # | ● |
| f. Verify solutions and determine the reasonableness of results | | | |
| i. In real-world situations | ● | ● | ● |
| ii. In abstract settings | △ | △ | ● |

● Subtopic can be assessed at this grade level.
△ Subtopic should not be assessed at this grade level.
# Subtopic may be introduced at a simple level (such as using a manipulative or pictorial model).

| Number Sense, Properties, and Operations | Grade 4 | Grade 8 | Grade 12 |
|---|---|---|---|
| 5. Apply ratios and proportional thinking in a variety of situations | | | |
| a. Use ratios to describe situations | # | ● | ● |
| b. Use proportions to model problems | △ | ● | ● |
| c. Use proportional thinking to solve problems (including rates, scaling, and similarity) | △ | ● | ● |
| d. Understand the meaning of percentage (including percentages greater than 100 and less than 1) | # | ● | ● |
| e. Solve problems involving percentages | △ | ● | ● |
| 6. Use elementary number theory | | | |
| a. Describe odd and even numbers and their characteristics | ● | ● | ● |
| b. Describe number patterns | # | ● | ● |
| c. Use factors and multiples to model and solve problems | △ | ● | ● |
| d. Describe prime numbers | △ | ● | ● |
| e. Use divisibility and remainders in problem settings (including simple modular arithmetic) | △ | # | ● |

● Subtopic can be assessed at this grade level.
△ Subtopic should not be assessed at this grade level.
# Subtopic may be introduced at a simple level (such as using a manipulative or pictorial model).

## Measurement

This strand focuses on an understanding of measurement and the use of numbers and measures to describe and compare mathematical and real-world objects. Students are asked to identify attributes, select appropriate units and tools, apply measurement concepts, and communicate measurement-related ideas.

Students should understand and be able to use the measurement attributes of length, mass/weight, capacity, time, money, and temperature. Students demonstrate their ability to extend basic concepts in applications involving, for example, perimeter, area, surface area, volume, and angle measure.

Students use measuring instruments and apply measurement concepts to solve problems. Due to the inherent imprecision of

measurement tools, it is important for students to recognize that measurement is an approximation.

When students use technology for calculations with imprecise measurements, errors are often carried or increased. Students should be assessed on their judgments about such answers.

The assessment focus at grade 4 is on time, money, temperature, length, perimeter, area, capacity, weight/mass, and angle measure. Although assessment at grades 8 and 12 continues to include these measurement concepts, the focus shifts to more complex measurement problems that involve volume or surface area or that require students to combine shapes, translate, and apply measures. Students in grades 8 and 12 should also solve problems involving proportional thinking (such as scale drawing or map reading) and do applications that involve the use of complex measurement formulas. When appropriate and possible, measurement is assessed with real measuring devices.

Items requiring straightforward computation with measures, especially those involving time and money, are included not in this content strand but in Number Sense, Properties, and Operations.

Applications involving measurement are a rich source of questions that assess the connections among number sense and operations, algebra, and geometry.

**NAEP Mathematics Content Strand 2**

| **Measurement** | **Grade** 4 | 8 | 12 |
|---|---|---|---|
| 1. Estimate the size of an object or compare objects with respect to a given attribute (such as length, area, capacity, volume, weight/mass) | ● | ● | ● |
| 2. Select and use appropriate measurement instruments (for example, manipulatives such as ruler, meter stick, protractor, thermometer, scales for weight or mass, gauges) | ● | ● | ● |

● Subtopic can be assessed at this grade level.
△ Subtopic should not be assessed at this grade level.
# Subtopic may be introduced at a simple level (such as using a manipulative or pictorial model).

| Measurement | Grade 4 | Grade 8 | Grade 12 |
|---|---|---|---|
| 3. Select and use appropriate units of measurement according to: | | | |
| a. Type of unit | ● | ● | ● |
| b. Size of unit | ● | ● | ● |
| 4. Estimate, calculate (using basic principles or formulas), or compare perimeter, area, volume, and surface area in meaningful contexts to solve mathematical and real-world problems | | | |
| a. Solve problems involving perimeter and area (such as triangles, quadrilaterals, other polygons, circles, combined forms) [Note: Grade 4 tasks use manipulatives] | # | ● | ● |
| b. Solve problems involving volume and surface area (such as rectangular solids, cylinders, cones, pyramids, prisms, combined forms) [Note: Grades 4 and 8 use manipulatives] | # | # | ● |
| 5. Apply given measurement formulas for perimeter, area, volume, and surface area in problem settings | △ | ● | ● |
| 6. Convert from one measurement to another within the same system (customary or metric) | △ | ● | ● |
| 7. Determine precision, accuracy, and error | | | |
| a. Apply significant digits in meaningful contexts | △ | ● | ● |
| b. Determine appropriate size of unit of measurement in problem situations | △ | ● | ● |
| c. Apply concepts of accuracy of measurement in problem situations | △ | ● | ● |
| d. Apply absolute and relative error in problem situations | △ | △ | ● |
| 8. Make and read scale drawings | △ | ● | ● |
| 9. Select appropriate methods of measurement (such as direct or indirect) | ● | ● | ● |
| 10. Apply the concept of rate to measurement situations | △ | ● | ● |

● Subtopic can be assessed at this grade level.
△ Subtopic should not be assessed at this grade level.
# Subtopic may be introduced at a simple level (such as using a manipulative or pictorial model).

## Geometry and Spatial Sense

Spatial sense must be an integral component of the study and assessment of geometry. Understanding spatial relationships allows students to use the dynamic nature of geometry to connect mathematics to their world.

This content strand extends well beyond low-level identification of geometric shapes into transformations and combinations of those shapes. Informal constructions and demonstrations (including drawing representations), along with their justifications, take precedence over more traditional types of compass-and-straightedge constructions and proofs. Although reasoning is addressed throughout the content areas, this strand addresses reasoning in formal and informal settings. The extension of proportional thinking to similar figures and indirect measurement is an important aspect of this strand.

In grade 4, students are expected to model properties of shapes under simple combinations and transformations and use mathematical communication skills to draw figures given a verbal description. In grade 8, students are expected to understand properties of angles and polygons and apply reasoning skills to make and validate conjectures about transformations and combinations of shapes. In grade 12, students are expected to demonstrate proficiency with transformational geometry and to apply concepts of proportional thinking to various geometric situations. They also have opportunities to demonstrate more sophisticated reasoning processes, and they are also expected to demonstrate various algebraic and geometric connections. The importance of these connections and their use in solving problems is indicated by the shifting emphasis in geometry to coordinate geometry, as described in chapter four.

## NAEP Mathematics Content Strand 3

| Geometry and Spatial Sense | Grade 4 | Grade 8 | Grade 12 |
|---|---|---|---|
| 1. Describe, visualize, draw, and construct geometric figures | | | |
| a. Draw or sketch a figure given a verbal description (open-ended items) | ● | ● | ● |
| b. Given a figure, write a verbal description of its geometric qualities | △ | ● | ● |
| 2. Investigate and predict results of combining, subdividing, and changing shapes (such as paper folding, dissecting, tiling, rearranging pieces of solids) | ● | ● | ● |
| 3. Identify the relationship (congruence, similarity) between a figure and its image under a transformation | | | |
| a. Use motion geometry (informal: lines of symmetry, flips, turns, slides) | ● | ● | ● |
| b. Use transformations (translations, rotations, reflections, dilations, symmetry) | | | |
| i. Synthetic | △ | # | ● |
| ii. Algebraic | △ | △ | ● |
| 4. Describe the intersection of two or more geometric figures | | | |
| a. Two dimensional | △ | ● | ● |
| b. Planar cross-section of a solid | △ | ● | ● |
| 5. Classify figures in terms of congruence and similarity, and informally apply these relationships using proportional reasoning where appropriate | △ | ● | ● |
| 6. Apply geometric properties and relationships in solving problems | | | |
| a. Use concepts of "between," "inside," "on," and "outside" | ● | ● | △ |
| b. Use the Pythagorean relationship to solve problems | △ | ● | ● |
| c. Apply properties of ratio and proportion with respect to similarity | △ | # | ● |
| d. Solve problems involving right triangle trigonometric applications | △ | △ | ● |

● Subtopic can be assessed at this grade level.
△ Subtopic should not be assessed at this grade level.
# Subtopic may be introduced at a simple level (such as using a manipulative or pictorial model).

| Geometry and Spatial Sense | Grade 4 | Grade 8 | Grade 12 |
|---|---|---|---|
| 7. Establish and explain relationships involving geometric concepts | | | |
| a. Make conjectures | ● | ● | ● |
| b. Validate and justify conclusions and generalizations | ● | ● | ● |
| c. Use informal induction and deduction | # | ● | ● |
| 8. Represent problem situations with geometric models and apply properties of figures in meaningful contexts to solve mathematical and real-world problems | ● | ● | ● |
| 9. Represent geometric figures and properties algebraically using coordinates and vectors | | | |
| a. Use properties of lines (including distance, midpoint, slope, parallelism, perpendicularity) to describe figures algebraically | △ | # | ● |
| b. Algebraically describe conic sections and their properties | △ | △ | ● |
| c. Use vectors in problem situations (addition, subtraction, scalar multiplication, dot product) | △ | △ | ● |

● Subtopic can be assessed at this grade level.
△ Subtopic should not be assessed at this grade level.
# Subtopic may be introduced at a simple level (such as using a manipulative or pictorial model).

## Data Analysis, Statistics, and Probability

Because of its fundamental role in making sense of the world, this content strand receives increased emphasis. The important skills of collecting, organizing, reading, representing, and interpreting data are assessed in various contexts to reflect the pervasive use of these skills in dealing with information. Statistics and statistical concepts extend these basic skills to include analyzing and interpreting increasingly sophisticated data. Dealing with uncertainty and making predictions about outcomes require an understanding of not only the meaning of basic probability concepts but also the application of those concepts in problem-solving and decisionmaking situations.

Questions emphasize appropriate methods of gathering data, the visual exploration of data, ways to represent data, and the development and evaluation of arguments based on data analysis. Students are expected to apply these ideas in increasingly sophisticated situations that require increasingly comprehensive analysis and decisionmaking.

In grade 4, students are expected to apply their understanding of number and quantity by solving problems involving data and to use data analysis to broaden their number sense. They are expected to be familiar with various graphs. They are asked to make predictions from data and explain their reasoning and to deal informally with measures of central tendency. Grade 4 students also are asked to use the basic concept of chance in meaningful contexts not involving the computation of probabilities.

Probabilistic thinking and various specialized graphs become increasingly important in grades 8 and 12. Students in grade 8 are expected to analyze statistical claims and design experiments, and they may use simulations to model real-world situations. They should have some understanding of sampling, and they should be asked to make predictions based on experiments or data. They will begin to use some formal terminology related to probability, data analysis, and statistics. By grade 8, students should be comfortable using various graphs to represent different types of data in different situations.

Students in grade 12 are expected to use a variety of statistical techniques to model situations and solve problems. Students at this level should apply concepts of probability to explore dependent and independent events, and they should be somewhat knowledgeable about conditional probability. They should be able to use formulas and more formal terminology to describe various situations. At this level, students should have a basic understanding of the use of mathematical equations and graphs to interpret data, including the use of curve fitting to match a set of data with an appropriate mathematical model.

## NAEP Mathematics Content Strand 4

| Data Analysis, Statistics, and Probability | Grade 4 | Grade 8 | Grade 12 |
|---|---|---|---|
| 1. Read, interpret, and make predictions using tables and graphs | | | |
| a. Read and interpret data | ● | ● | ● |
| b. Solve problems by estimating and computing with data | ● | ● | ● |
| c. Interpolate or extrapolate from data | △ | ● | ● |
| 2. Organize and display data and make inferences | | | |
| a. Use tables, histograms (bar graphs), pictograms, and line graphs | ● | ● | ● |
| b. Use circle graphs and scattergrams | △ | ● | ● |
| c. Use stem-and-leaf plots and box-and-whisker plots | △ | ● | ● |
| d. Make decisions about outliers | △ | ● | ● |
| 3. Understand and apply sampling, randomness, and bias in data collection | | | |
| a. Given a situation, identify sources of sampling error | △ | ● | ● |
| b. Describe a procedure for selecting an unbiased sample | △ | ● | ● |
| c. Make generalizations based on sample results | △ | ● | ● |
| 4. Describe measures of central tendency and dispersion in real-world situations | # | ● | ● |
| 5. Use measures of central tendency, correlation, dispersion, and shapes of distributions to describe statistical relationships | | | |
| a. Use standard deviation and variance | △ | △ | ● |
| b. Use the standard normal distribution | △ | △ | ● |
| c. Make predictions and decisions involving correlation | △ | △ | ● |
| 6. Understand and reason about the use and misuse of statistics in our society | | | |
| a. Given certain situations and reported results, identify faulty arguments or misleading presentations of the data | # | ● | ● |
| b. Appropriately apply statistics to real-world situations | # | ● | ● |
| 7. Fit a line or curve to a set of data and use this line or curve to make predictions about the data, using frequency distributions where appropriate | △ | △ | ● |

● Subtopic can be assessed at this grade level.
△ Subtopic should not be assessed at this grade level.
# Subtopic may be introduced at a simple level (such as using a manipulative or pictorial model).

| Data Analysis, Statistics, and Probability | Grade 4 | Grade 8 | Grade 12 |
|---|---|---|---|
| 8. Design a statistical experiment to study a problem and communicate the outcomes | △ | ● | ● |
| 9. Use basic concepts, trees, and formulas for combinations, permutations, and other counting techniques to determine the number of ways an event can occur | △ | ● | ● |
| 10. Determine the probability of a simple event | | | |
| a. Estimate probabilities by use of simulations | △ | ● | ● |
| b. Use sample spaces and the definition of probability to describe events | ● | ● | ● |
| c. Describe and make predictions about expected outcomes | △ | ● | ● |
| 11. Apply the basic concept of probability to real-world situations | | | |
| a. Use probabilistic thinking informally | ● | ● | ● |
| b. Use probability related to independent and dependent events | △ | ● | ● |
| c. Use probability related to simple and compound events | △ | △ | ● |
| d. Use conditional probability | △ | △ | ● |

● Subtopic can be assessed at this grade level.
△ Subtopic should not be assessed at this grade level.
# Subtopic may be introduced at a simple level (such as using a manipulative or pictorial model).

## Algebra and Functions

This strand extends from work with simple patterns at grade 4 to basic algebra concepts at grade 8 and sophisticated analysis at grade 12; it involves not only algebra but also precalculus and some topics from discrete mathematics. Algebraic concepts are developed throughout the grades, emphasizing informal modeling at the elementary level and functions at the secondary level. Students are expected to use algebraic notation and thinking in meaningful contexts to solve mathematical and real-world problems, specifically addressing an increasing understanding of the use of functions (including algebraic and geometric) as a representational tool.

The assessment at all levels includes the use of open sentences and equations as representational tools. Students are expected to

use equivalent representations to transform and solve number sentences and equations of increasing levels of complexity.

The grade 4 assessment involves the informal demonstration of students' abilities to generalize from patterns and justify their generalizations. Students are expected to translate mathematical representations, use simple equations, and demonstrate basic graphing.

The grade 8 assessment includes more algebraic notation, stressing the meaning of variables and an informal understanding of the use of symbolic representations in problem-solving contexts. Students at this level are asked to use variables to represent a rule underlying a pattern. They should have a beginning understanding of equations as a modeling tool, and they should solve simple equations and inequalities through various methods, including both graphical and basic algebraic methods. Students should begin to use basic concepts of functions to describe relationships.

In grade 12, students are expected to be adept at appropriately choosing and applying a rich set of representational tools in various problem-solving situations. They should have an understanding of basic algebraic notation and terminology as they relate to representations of mathematical and real-world problem situations. Students should be able to use functions to represent and describe relationships.

**NAEP Mathematics Content Strand 5**

| **Algebra and Functions** | **Grade 4** | **8** | **12** |
|---|---|---|---|
| 1. Describe, extend, interpolate, transform, and create a wide variety of patterns and functional relationships | | | |
| a. Recognize patterns and sequences | ● | ● | ● |
| b. Extend a pattern or functional relationship | ● | ● | ● |
| c. Given a verbal description, extend or interpolate with a pattern (complete a missing term) | △ | ● | ● |

| | |
|---|---|
| ● | Subtopic can be assessed at this grade level. |
| △ | Subtopic should not be assessed at this grade level. |
| # | Subtopic may be introduced at a simple level (such as using a manipulative or pictorial model). |

| Algebra and Functions | Grade 4 | Grade 8 | Grade 12 |
|---|---|---|---|
| d. Translate patterns from one context to another | # | ● | ● |
| e. Create an example of a pattern or functional relationship | ● | ● | ● |
| f. Understand and apply the concept of a variable | # | ● | ● |
| 2. Use multiple representations for situations to translate among diagrams, models, and symbolic expressions | ● | ● | ● |
| 3. Use number lines and rectangular coordinate systems as representational tools | | | |
| a. Identify or graph sets of points on a number line or in a rectangular coordinate system | ● | ● | ● |
| b. Identify or graph sets of points in a polar coordinate system | △ | △ | ● |
| c. Work with applications using coordinates | △ | ● | ● |
| d. Transform the graph of a function | △ | # | ● |
| 4. Represent and describe solutions to linear equations and inequalities to solve mathematical and real-world problems | | | |
| a. Provide solution sets of whole numbers | ● | ● | ● |
| b. Provide solution sets of real numbers | # | ● | ● |
| 5. Interpret contextual situations and perform algebraic operations on real numbers and algebraic expressions to solve mathematical and real-world problems | | | |
| a. Perform basic operations, using appropriate tools, on real numbers in meaningful contexts (including grouping and order of multiple operations involving basic operations, exponents, and roots) | △ | ● | ● |
| b. Solve problems involving substitution in expressions and formulas | △ | ● | ● |
| c. Solve meaningful problems involving a formula with one variable | △ | ● | ● |
| d. Use equivalent forms to solve problems | △ | ● | ● |
| 6. Solve systems of equations and inequalities using appropriate methods | | | |
| a. Solve systems graphically | △ | ● | ● |
| b. Solve systems algebraically | △ | △ | ● |
| c. Solve systems using matrices | △ | △ | ● |

● Subtopic can be assessed at this grade level.
△ Subtopic should not be assessed at this grade level.
# Subtopic may be introduced at a simple level (such as using a manipulative or pictorial model).

| Algebra and Functions | Grade 4 | Grade 8 | Grade 12 |
|---|---|---|---|
| 7. Use mathematical reasoning | | | |
| a. Make conjectures | ● | ● | ● |
| b. Validate and justify conclusions and generalizations | ● | ● | ● |
| c. Use informal induction and deduction | # | ● | ● |
| 8. Represent problem situations with discrete structures | | | |
| a. Use finite graphs and matrices | △ | # | ● |
| b. Use sequences and series | △ | △ | ● |
| c. Use recursive relations (including numerical and graphical iteration and finite differences) | △ | △ | ● |
| 9. Solve polynomial equations with real and complex roots using a variety of algebraic and graphical methods and using appropriate tools | △ | △ | ● |
| 10. Approximate solutions of equations (bisection, sign changes, and successive approximations) | △ | # | ● |
| 11. Use appropriate notation and terminology to describe functions and their properties (including domain, range, function composition, and inverses) | △ | △ | ● |
| 12. Compare and apply the numerical, symbolic, and graphical properties of a variety of functions and families of functions, examining general parameters and their effect on curve shape | △ | # | ● |
| 13. Apply function concepts to model and deal with real-world situations | △ | # | ● |
| 14. Use trigonometry | | | |
| a. Use triangle trigonometry to model problem situations | △ | △ | ● |
| b. Use trigonometric and circular functions to model real-world phenomena | △ | △ | ● |
| c. Apply concepts of trigonometry to solve real-world problems | △ | △ | ● |

● Subtopic can be assessed at this grade level.
△ Subtopic should not be assessed at this grade level.
# Subtopic may be introduced at a simple level (such as using a manipulative or pictorial model).

# Chapter Four

## Cognitive Abilities

Although NAEP was designed to monitor, assess, and report student achievement nationally, an inevitable effect of this monitoring and reporting is improvement in mathematics learning. If real change in the mathematics curriculum is to take place, the manner in which assessment is conducted will also have to change. Assessment activities often are the primary sources from which students discern what teachers really value and what teachers really want them to know.

### Mathematical Power

Mathematical power is characterized as a student's overall ability to gather and use mathematical knowledge through exploring, conjecturing, and reasoning logically; solving nonroutine problems; communicating about and through mathematics; and connecting mathematical ideas in one context with mathematical ideas in another context or with ideas from another discipline in the same or related contexts.

Assessing a student's mathematical power requires many different indicators over time. As power develops beyond the general mathematical abilities of conceptual understanding, procedural knowledge, and problem solving, it is important to ensure that students are assessed on their ability to *reason* in mathematical situations, *communicate* perceptions and conclusions drawn from a mathematical context, and *connect* the mathematical nature of a situation with related mathematical knowledge and information gained from other disciplines or through observation.

It is the total interaction of all of these abilities that defines a student's overall mathematical power at a given time. The mental skills of reasoning, communicating, and connecting are the foundation of each content strand and each mathematical ability featured in previous NAEP assessments. These relationships, discussed in

chapter two, indicate the multidimensional nature of mathematical power.

Mathematical power can be viewed from various perspectives. Students may encounter a new problem in an old context or an old problem in a new context. When first attempts to solve a problem fail, the student may reexamine the information, rework it, and then reapply it to the situation in a more productive fashion. The process of revising an approach to a problem based on reasoning, gathering new information, and making connections with other ideas is a dynamic ability. This feature of mathematical power can be viewed through student performance within a particular content strand at the conceptual, procedural, and problem-solving levels of ability. Similarly, a particular concept, procedure, or problem context might be viewed across strands. In the latter case, families of items are particularly helpful in assessment. The use of calculators enables students to quickly pursue alternative paths and determine whether they provide fruitful new information or reconfirm judgments made through other approaches.

Students demonstrate their mathematical power by formulating problem-solving and reasoning strategies in situations involving a multitude of possibilities. It is here that the recommendation that students experience a number of extended open-ended items requiring construction of responses is important. Through a student's report of his or her thinking, questions of the relevance of the approach, the nature of reasoning, and the ability to solve problems become less inferential and more conclusively based on evidence. This is especially true when the collected evidence includes the communication of a student's approach and when partial credit for student efforts is awarded in the scoring of an item.

Finally, mathematical power is a function of students' prior knowledge and experience and the ability to connect that knowledge in productive ways to new contexts. This aspect of power can be measured with multiple-choice items and through analysis of the ways in which students develop their responses to the constructed-response items on the assessment.

Information related to these features of students' development is as difficult to isolate and statistically extract from the data as the

mathematical abilities featured in the past NAEP assessments in mathematics. However, they are important aspects of the mathematical development of students. As such, the three features of mathematical power (reasoning, communication, and connections) will be used as underlying threads for item construction and overall test design. For the mathematics assessment, these threads may not be specifically reported, although they will be represented in the overall way the assessment is conceived and developed.

## Mathematical Abilities

As previously discussed, the general mental abilities associated with mathematics and targeted in past NAEP assessments are conceptual understanding, procedural knowledge, and problem solving. These three areas are specifically identified as primary foci for assessment, and they received focal attention in the design of the 1990 and 1992 assessments. Conceptual understanding can be viewed simply as a measure of a student's "knowing that" or "knowing about," whereas procedural knowledge can be viewed as a student's "knowing how." These two abilities are the foundation for recognizing and understanding a problem, formulating a plan to solve the problem, arriving at a solution to the problem, and reflecting on the solution. The later stages can be thought of as facets of problem solving.

However, as recommended in chapter one, the role of these dimensions of students' mathematical power in the new assessment should change from one of a direct matrix feature to one of a design characteristic that assists in providing balance to the overall assessment. The NAEP design for the mathematics assessment should certainly continue to focus on conceptual understanding, procedural knowledge, and problem solving in bringing some balance to the assessments for grades 4, 8, and 12. In particular, it is recommended that at least one-third of the items for each grade level measure conceptual understanding, procedural knowledge, and problem solving.

As with the mathematical content strands, mathematical abilities are not separate and distinct factors of an individual's ways of thinking about a mathematical situation. These abilities are, rather,

descriptions of the ways in which information is structured for instruction and the ways in which students manipulate, reason with, or communicate their mathematical ideas. Consequently, no unanimous agreement exists among educators about what constitutes a conceptual, a procedural, or a problem-solving item. What can be classified are the actions a student is likely to undertake in processing information and providing a satisfactory response. Thus, within the content strands, assessment tasks are classified according to the ability categories they most closely represent in terms of the type of processing they are expected to require. Furthermore, the mathematical power features of reasoning, communication, and connections are woven through the specifications to provide an added level of richness to the assessment tasks.

The following discussions of conceptual understanding, procedural knowledge, and problem solving illustrate the primary features the NAEP assessment should employ to capture features of cognitive activities that combine to empower a student in mathematical situations.

## Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can recognize, label, and generate examples and nonexamples of concepts; use and interrelate models, diagrams, manipulatives, and varied representations of concepts; identify and apply principles (that is, valid statements generalizing relationships among concepts in conditional form); know and apply facts and definitions; compare, contrast, and integrate related concepts and principles to extend the nature of concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts; or interpret the assumptions and relations involving concepts in mathematical settings.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either. Students demonstrate conceptual understanding when they produce examples or common or unique representations, or when they manipulate central ideas about a concept in various ways.

## Procedural Knowledge

Students demonstrate procedural knowledge in mathematics when they select and apply appropriate procedures correctly; verify or justify the correctness of a procedure using concrete models or symbolic methods; or extend or modify procedures to deal with factors inherent in problem settings.

Procedural knowledge includes the various numerical algorithms in mathematics that have been created as tools to meet specific needs efficiently. Procedural knowledge also encompasses the abilities to read and produce graphs and tables, execute geometric constructions, and perform noncomputational skills such as rounding and ordering. These latter activities can be differentiated from conceptual understanding by the task context or presumed student background—that is, an assumption that the student has the conceptual understanding of a representation and can apply it as a tool to create a product or to achieve a numerical result. In these settings, the assessment question is how well the student executed a procedure or selected the appropriate procedure to perform a given task.

Procedural knowledge is often reflected in a student's ability to connect an algorithmic process with a given problem situation, employ that algorithm correctly, and communicate the results of the algorithm in the context of the problem setting. Procedural understanding also encompasses a student's ability to reason through a situation, describing why a particular procedure will solve a problem in the context described.

## Problem Solving

In problem solving, students are required to use their accumulated knowledge of mathematics in new situations. Problem solving requires students to recognize and formulate problems; determine the sufficiency and consistency of data; use strategies, data, models, and relevant mathematics; generate, extend, and modify procedures; use reasoning (spatial, inductive, deductive, statistical, or proportional) in new settings; and judge the reasonableness and correctness of solutions. Problem-solving situations require students to connect all of their mathematical knowledge of concepts, procedures, reasoning, and communication/representational skills in confronting new situations. As such, these situations are perhaps the most accurate measures of students' proficiency in mathematics.

# Chapter Five

## Item Types

Central to the development of the NAEP assessment in mathematics is the careful selection of test questions or items to measure the content objectives and cognitive areas. The NAEP mathematics assessment consists of multiple-choice, short constructed-response, and extended constructed-response items. Examples of NAEP mathematics items for grades 4, 8, and 12 are provided below. Please refer to the NAEP Web site at *www.nces.ed.gov/nationsreportcard* for additional items, scoring rubrics, performance data, and sample student responses.

## Multiple-Choice Items

### Grade 4

$503 - 207 =$

A. 206
*B. 296
C. 304
D. 396

**[Percent correct: 53%]**

*N* stands for the number of stamps John had. He gave 12 stamps to his sister. Which expression tells how many stamps John has now?

A. $N + 12$
*B. $N - 12$
C. $12 - N$
D. $12 \times N$

**[Percent correct: 67%]**

In a bag of marbles, 1/2 are red, 1/4 are blue, 1/6 are green, and 1/12 are yellow. If a marble is taken from the bag without looking, it is most likely to be

*A. red
B. blue
C. green
D. yellow

**[Percent correct: 25%]**

## Grade 8

If $\frac{2}{25} = \frac{n}{500}$, then $n$ =

A. 10
B. 20
C. 30
*D. 40
E. 50

**[Percent correct: 48%]**

Which of the following ordered pairs $(x, y)$ is a solution to the equation $2x - 3y = 6$?

A. (6, 3)
*B. (3, 0)
C. (3, 2)
D. (2, 3)
E. (0, 3)

**[Percent correct: 41%]**

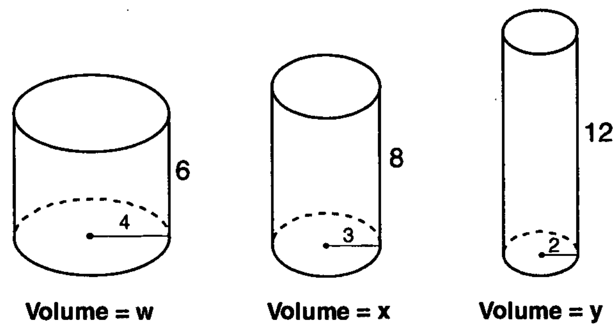
How many hours are equal to 150 minutes?

A. $1\frac{1}{2}$
B. $2\frac{1}{4}$
C. $2\frac{1}{3}$
*D. $2\frac{1}{2}$
E. $2\frac{5}{6}$

**[Percent correct: 58%]**

(Note: Fractions appeared with horizontal fraction bars on the exam.)

## Grade 12



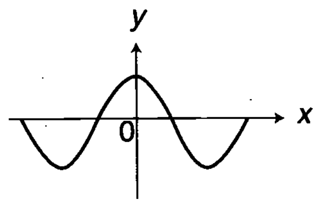

In the figures above, the radius and height of each right circular cylinder are given. If w, x, and y represent the volume of the cylinders, which of the following statements is true?

A. $y = w = x$
*B. $y < x < w$
C. $y < w < x$
D. $w < y < x$
E. $w < x < y$

**[Percent correct: 30%]**

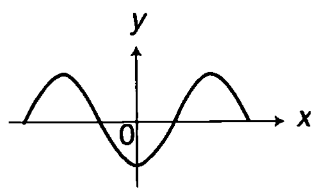
48

## Grade 12

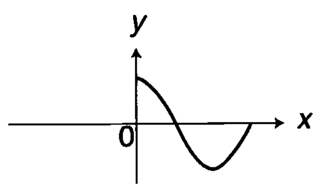



The figure above shows the graph of $y = f(x)$. Which of the following could be the graph of $y = |f(x)|$?

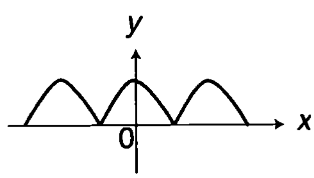
A.



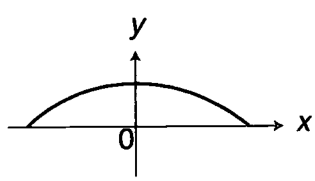

B.



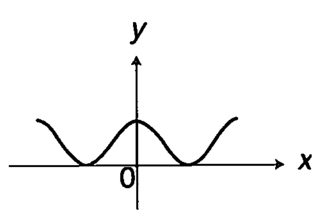

*C.




D.

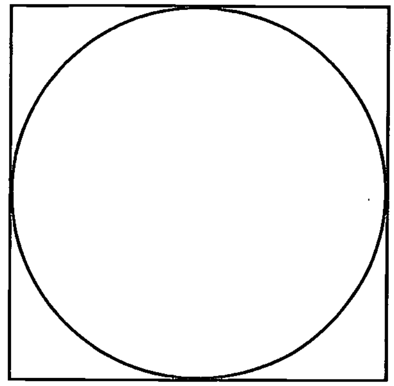



E.




**[Percent correct: 20%]**

## Grade 12



The length of a side of the square above is 6. What is the length of the radius of the circle?

A. 2
*B. 3
C. 4
D. 6
E. 8

**[Percent correct: 70%]**

50

## Short Constructed-Response Items

To provide more reliable and valid opportunities for extrapolating about students' approaches to problems, recent NAEP assessments have included items that are often referred to as constructed response or open ended. These short-answer items require students to give either a numerical result or the correct name or classification for a group of mathematical objects, draw an example of a given concept, or write a brief explanation for a given result.

### Grade 4

Ms. Hernandez formed teams of 8 students each from the 34 students in her class. She formed as many teams as possible, and the students left over were substitutes.

How many students were substitutes?

Answer: ____________________

**[Percent correct: 39%]**

| **Scoring Guide** |
|---|
| **Score and Description**<br>**Correct**<br>2 |
| **Incorrect**<br>Incorrect response |

## Grade 4

How many fourths make a whole?

Answer: ____________________

**[Percent correct: 50%]**

| **Scoring Guide** |
| --- |
| **Score and Description** |
| **Correct** |
| Correct Response |
| 4, or four fourths, or 4 fourths, etc. |
| **Incorrect** |
| Incorrect response |

## Grade 8

From any vertex of a 4-sided polygon, 1 diagonal can be drawn.

From any vertex of a 5-sided polygon, 2 diagonals can be drawn.

From any vertex of a 6-sided polygon, 3 diagonals can be drawn.

From any vertex of a 7-sided polygon, 4 diagonals can be drawn.

How many diagonals can be drawn from any vertex of a 20-sided polygon?

Answer: ____________________

**[Percent correct: 54%]**

| **Scoring Guide** |
|---|
| **Solution:**<br>17 |

In this question a student needed to demonstrate an understanding of diagonals of polygons. A diagonal of a polygon is a segment that joins two nonadjacent vertices (a vertex is a common endpoint of two sides of the polygon). To answer the question it was expected that a student would observe that the number of diagonals from any vertex is 3 less than the number of sides. That is, from any vertex of a convex polygon a diagonal can be drawn to any of the other vertices of that polygon except the two adjacent vertices. For a 20-sided polygon, the answer is $20 - 3 = 17$.

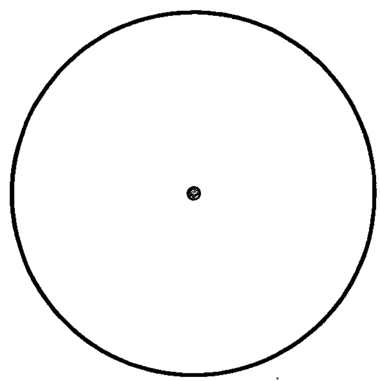
| **Score and Description** |
|---|
| **Correct**<br>Correct response (17) |
| **Incorrect**<br>Any incorrect response |

## Grade 8

### Hair Color Survey Results

| Color of Hair | Percentage |
|---|---|
| Blond | 17 |
| Brown | 50 |
| Black | 33 |
| Total | 100 |

The table above shows the results of a survey of hair color. On the circle below, make a circle graph to illustrate the data in the table. Label each part of the circle graph with the correct hair color.

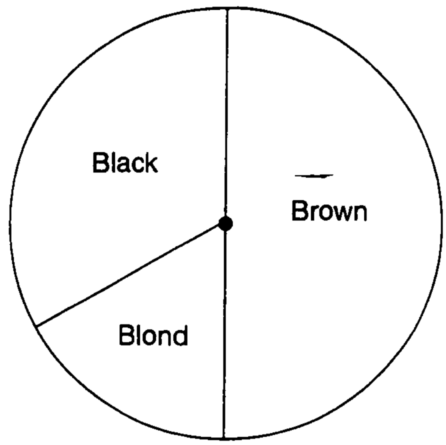


**[Percent correct: 72%]**

54

## Scoring Guide

**Score and Description**




**Correct**

The brown region should be about ½ of the circle.

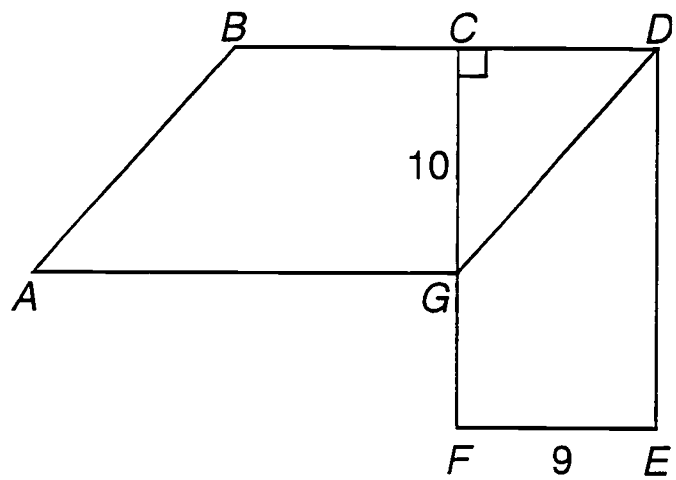
The blond region should be about ½ the black region.

All three regions must be labeled with the correct colors or with the correct percents.

**Incorrect**

Incorrect answer

## Grade 12




In the figure above, *ABDG* is a parallelogram and *CDEF* is a rectangle.

If $EF = 9$ and $CG = 10$, what is *AB* to the nearest hundredth?

Answer: ____________________

(Students had access to a calculator.)

**[Percent correct: 21%]**

| **Scoring Guide** |
|---|
| **Score and Description** |
| **Correct** |
| 13.45 |
| Solution: |
| $FE = CD = 9$ |
| $AB = DG = \sqrt{10^2 + 9^2}$ |
| $= \sqrt{181}$ |
| $= 13.45$ |
| **Incorrect #1** |
| Incorrect answer other than $\sqrt{181}$ |
| **Incorrect #2** |
| $\sqrt{181}$ |

## Extended Constructed-Response Items

Extended constructed-response items require students to consider a situation that demands more than a numerical or short verbal response. These items require the student to carefully consider a problem within or across the content strands, understand what is required to "solve" the problem, choose a plan of attack, carry out the attack, and interpret the solution in terms of the original problem. The response mode requires that students provide evidence of their work on some aspects of the problem-solving process and communicate their decisionmaking steps in the context of the problem.

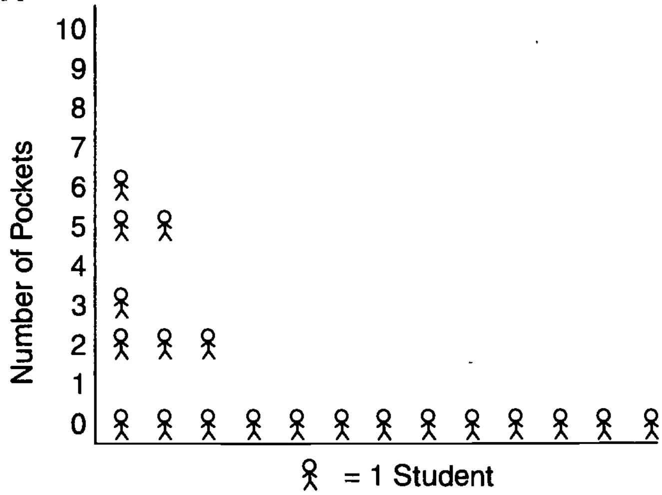
### Grade 4

Think carefully about the following question. Write a complete answer. You may use drawings, words, and numbers to explain your answer. Be sure to show all of your work.

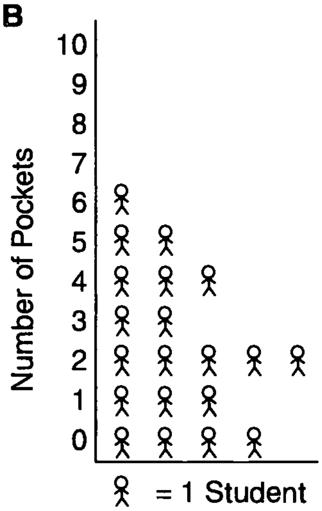
There are 20 students in Mr. Pang's class. On Tuesday, <u>most</u> of the students in the class said they had pockets in the clothes they were wearing.

# Grade 4

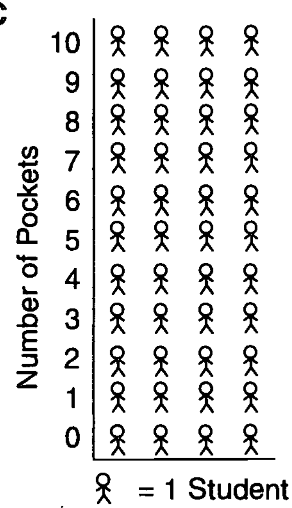
A




B




C




Which of the graphs most likely shows the number of pockets that each child had?

Explain why you chose that graph.

Explain why you did not choose the other graphs.

## Scoring Guide
### Solution:

Graph B because it had 20 students and most of the students had pockets.

It could not be graph A because most of the students should have pockets.

## Grade 4

It could not be graph C, since there are more than 20 students shown.

OR

It is not likely that there would be the same number of students for each number of pockets.

OR

Most clothes don't have 10 pockets.

| Score and Description |
|---|
| **Extended**<br><br>The student chooses graph B and gives a good explanation why it should be B and explains why it can't be A or C. The explanation must deal with both the number of students in the class and the fact that most of them have pockets. These explanations may occur in either response. |
| **Satisfactory**<br><br>The student chooses graph B and gives a good explanation (which includes the fact that graph B has 20 students and most of the students have pockets) but does not mention the other graphs.<br><br>OR<br><br>The student gives a good explanation why it cannot be A or C but does not give a good explanation of why it is B. (See examples in solution above for explanations.) |

## Grade 4

| |
|---|
| **Partial**<br><br>The student chooses graph B but does not give an adequate, relevant explanation.<br><br>OR<br><br>The student chooses B but gives no explanation why; student explains why it's not C or why it's not A. |
| **Minimal**<br><br>The student chooses graph B with no explanation or a weak, nonrelevant (e.g., because it made sense) explanation.<br><br>OR<br><br>The student chooses A or C with an explanation that shows some understanding. |
| **Incorrect/Off Task**<br><br>The work is completely incorrect, irrelevant, or off task.<br><br>OR<br><br>The student answers A or C with no explanation. |

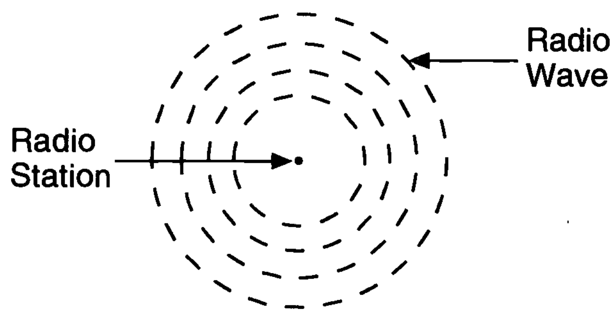
**[Extended 3%, Satisfactory 7%, Partial 15%, Minimal 23%]**

## Grade 8

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

Radio station KMAT in Math City is 200 miles from radio station KGEO in Geometry City. Highway 7, a straight road, connects the two cities.

KMAT broadcasts can be received up to 150 miles in all directions from the station and KGEO broadcasts can be received up to 125 miles in all directions. Radio waves travel from each radio station through the air, as represented above.




On the next page, draw a diagram that shows the following:

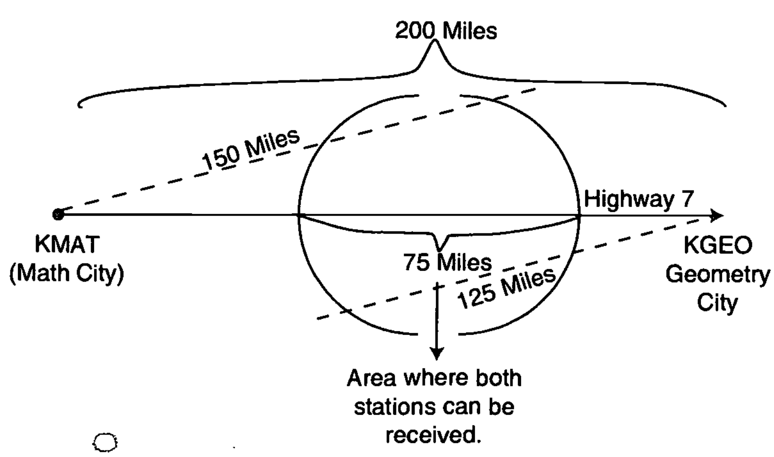
- Highway 7.
- The location of the two radio stations.
- The part of Highway 7 where both radio stations can be received.

Be sure to label the distances along the highway and the length in miles of the part of the highway where both stations can be received.

### Scoring Guide

**Solution:**

There is a 75-mile part of Highway 7 that is within both broadcast areas. It starts 75 miles outside Math City and ends 150 miles outside Math City.




61

## Grade 8

| Score and Description |
|---|
| **Extended**<br>Correct answer. (75 miles must be stated.) |
| **Satisfactory**<br>Map with cities or stations and 200 miles labeled (or a clear and correct application of scale) and identifies common broadcast area on Highway 7 but omits length of common area. |
| **Partial**<br>Map with cities or stations and 200 miles labeled (or some attempt at using a scale): the highway should be shown as straight, and identifies incorrect common broadcast area (e.g. not on Highway 7) or insufficiently identifies an area. (Insufficiently means that there is not enough information labeled to determine the length of the common broadcast area.) Bounds of common area may or may not be labeled. |
| **Minimal**<br>Map with cities or stations and 200 miles labeled (or some attempt to use a scale). Highway should be shown as straight. There is no indication of how student determined common broadcast area. (It may, for example, be represented as a single point or not at all.)<br>OR<br>Map that uses some but not all of given information with no indication of how common broadcast area was determined. |
| **Incorrect/Off Task**<br>The work is completely incorrect, irrelevant, or off task. |

**Note:**

A student's map must include enough information concerning the lengths given in the question to justify that 75 miles is the common broadcast area. If pertinent information concerning lengths is missing, the maximum score that can be awarded is Partial.

**[Extended 1%, Satisfactory 4%, Partial 13%, Minimal 22%]**

## Grade 12

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. You answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

The table below shows the daily attendance at two movie theaters for 5 days and the mean (average) and the median attendance.

| | Theater A | Theater B |
|---|---|---|
| Day 1 | 100 | 72 |
| Day 2 | 87 | 97 |
| Day 3 | 90 | 70 |
| Day 4 | 10 | 71 |
| Day 5 | 91 | 100 |
| Mean (average) | 75.6 | 82 |
| Median | 90 | 72 |

(a) Which statistic, the mean or the median, would you use to describe the typical daily attendance for the 5 days at Theater A? Justify your answer.

(b) Which statistic, the mean or the median, would you use to describe the typical daily attendance for the 5 days at Theater B? Justify your answer.

## Scoring Guide
### Solution:

Selects and provides appropriate explanation for why the mean is a better measure for the typical attendance for Theater B and the median is the better measure for Theater A.

An explanation for Theater A should include the idea that the attendance on day 4 is much different than the attendance numbers for any other days for Theater A.

## Grade 12

An appropriate explanation for Theater B should include the following ideas:

- There are two clusters of data.
- The median is representative of only one of the clusters while the mean is representative of both.

OR

- a justification that conveys the idea that 82 is a better indicator of where the "center" of the 5 data points is located

### Scoring Guide

In this question, a student has to look at the data and determine which measure, the median or the mean, would best describe the typical daily attendance at each theater. A student has to have an understanding of the meaning of mean and median in order to provide a correct answer and explanation. For full credit, a student has to answer the median for part a and include an explanation that would include that day 4's attendance is significantly different than the rest of the days and the mean for part b with an explanation that shows an understanding that the mean is a better indicator because all of the attendance numbers for Theater B are clustered. Varying levels of partial credit (satisfactory, partial, and minimal) could be earned depending on how well the student reasons and communicates the correct answer.

## Grade 12

| Score and Description |
|---|
| **Extended**<br><br>Indicates the better measure for each theater and gives a complete explanation for each measure. |
| **Satisfactory**<br><br>Indicates the better measure for each theater and gives a complete explanation for one measure. |
| **Partial**<br><br>Indicates mean for Theater B and median for Theater A with either no explanation or an incomplete explanation.<br><br>OR<br><br>The student selects the better measure for one theater and gives an appropriate explanation. |
| **Minimal**<br><br>Indicates the mean for Theater B with no explanation or an incomplete explanation.<br><br>OR<br><br>The student indicates the median for Theater A with no explanation or an incomplete explanation. |
| **Incorrect**<br><br>Incorrect response |

**[Extended 1%, Satisfactory 3%, Partial 10%, Minimal 28%]**

# References

Braswell, J.S., Lutkus, A.D., et.al. 2001. *The Nation's Report Card: Mathematics 2000.* Washington, DC: National Center for Education Statistics.

Reese, C.M., Miller, K.S., et.al. 1997. *NAEP 1996 Mathematics Report Card for the Nation and the States.* Washington, DC: National Center for Education Statistics.

Romberg, T.A., Wilson, L.D., Smith, M.E., and Smith, S.Z. 1991. *Improving Mathematical Performance: Reflections and Suggestions Based on the Results of NAEP's 1990 Twelfth-Grade Assessment.* Madison, WI: National Center for Research in Mathematical Sciences Education.

Silver, E.A., Kenney, P.A., and Salmon-Cox, L. 1991. *The Content and Curricular Validity of the 1990 NAEP Mathematics Items: A Retrospective Analysis.* Pittsburgh, PA: Learning Research and Development Center, University of Pittsburgh.

# Appendix A

## NAEP Mathematics Achievement Level Descriptions

# NAEP Mathematics Achievement Levels—Grade 4

**Basic** Fourth-grade students performing at the Basic level should show some evidence of understanding the mathematical concepts and procedures in the five NAEP content strands.

Fourth graders performing at the Basic level should be able to estimate and use basic facts to perform simple computations with whole numbers, show some understanding of fractions and decimals, and solve some simple real-world problems in all NAEP content areas. Students at this level should be able to use—although not always accurately—four-function calculators, rulers, and geometric shapes. Their written responses are often minimal and presented without supporting information.

**Proficient** Fourth-grade students performing at the Proficient level should consistently apply integrated procedural knowledge and conceptual understanding to problem solving in the five NAEP content strands.

Fourth-graders performing at the Proficient level should be able to use whole numbers to estimate, compute, and determine whether results are reasonable. They should have a conceptual understanding of fractions and decimals; be able to solve real-world problems in all NAEP content areas; and use four-function calculators, rulers, and geometric shapes appropriately. Students performing at the Proficient level should employ problem-solving strategies such as identifying and using appropriate information. Their written solutions should be organized and presented both with supporting information and with explanations of how they were achieved.

**Advanced** Fourth-grade students performing at the Advanced level should apply integrated procedural knowledge and conceptual understanding to complex and nonroutine real-world problem solving in the five NAEP content strands.

Fourth graders performing at the Advanced level should be able to solve complex nonroutine real-word problems in all NAEP content strands. They should display mastery in the use of four-function calculators, rulers, and geometric shapes. The students are expected to draw logical conclusions and justify answers and solution processes by explaining why, as well as how, they were achieved. They should go beyond the obvious in their interpretations and be able to communicate their thoughts clearly and concisely.

# NAEP Mathematics Achievement Levels—Grade 8

**Basic** Eighth-grade students performing at the Basic level should exhibit evidence of conceptual and procedural understanding in the five NAEP content strands. This level of performance signifies an understanding of arithmetic operations—including estimation—on whole numbers, decimals, fractions, and percents.

Eighth graders performing at the Basic level should complete problems correctly with the help of structural prompts such as diagrams, charts, and graphs. They should be able to solve problems in all NAEP content strands through the appropriate selection and use of strategies and technological tools—including calculators, computers, and geometric shapes. Students at this level also should be able to use fundamental algebraic and informal geometric concepts in problem solving.

As they approach the proficient level, students at the Basic level should be able to determine which of the available data are necessary and sufficient for correct solutions and use them in problem solving. However, these eighth graders show limited skill in communicating mathematically.

**Proficient** Eighth-grade students performing at the Proficient level should apply mathematical concepts and procedures consistently to complex problems in the five NAEP content strands.

Eighth graders performing at the Proficient level should be able to conjecture, defend their ideas, and give supporting examples. They should understand the connections among fractions, percents, decimals, and other mathematical topics such as algebra and functions. Students at this level are expected to have a thorough

understanding of basic-level arithmetic operations—an understanding sufficient for problem solving in practical situations.

Quantity and spatial relationships in problem solving and reasoning should be familiar to them, and they should be able to convey underlying reasoning skills beyond the level of arithmetic. They should be able to compare and contrast mathematical ideas and generate their own examples. These students should make inferences from data and graphs, apply properties of informal geometry, and accurately use the tools of technology. Students at this level should understand the process of gathering and organizing data and be able to calculate, evaluate, and communicate results within the domain of statistics and probability.

**Advanced** Eighth-grade students performing at the Advanced level should be able to reach beyond the recognition, identification, and application of mathematical rules to generalize and synthesize concepts and principles in the five NAEP content strands.

Eighth graders performing at the Advanced level should be able to probe examples and counterexamples to shape generalizations from which they can develop models.

Eighth graders performing at the Advanced level should use number sense and geometric awareness to consider the reasonableness of an answer. They are expected to use abstract thinking to create unique problem-solving techniques and explain the reasoning processes underlying their conclusions.

# NAEP Mathematics Achievement Levels—Grade 12

**Basic** Twelfth-grade students performing at the Basic level should demonstrate procedural and conceptual knowledge in solving problems in the five NAEP content strands.

Twelfth-grade students performing at the Basic level should be able to use estimation to verify solutions and determine the reasonableness of results as applied to real-world problems. Twelfth graders performing at the Basic level should recognize relationships presented in verbal, algebraic, tabular, and graphical forms, and demonstrate knowledge of geometric relationships and corresponding measurement skills.

They should be able to apply statistical reasoning in the organization and display of data and in reading tables and graphs. They should also be able to generalize from patterns and examples in the areas of algebra, geometry, and statistics. At this level, they should use correct mathematical language and symbols to communicate mathematical relationships and reasoning processes and use calculators appropriately to solve problems.

**Proficient** Twelfth-grade students performing at the Proficient level should consistently integrate mathematical concepts and procedures into the solutions of more complex problems in the five NAEP content strands.

Twelfth graders performing at the Proficient level should demonstrate an understanding of algebraic, statistical, geometric, and spatial reasoning. They should be able to perform algebraic operations involving polynomials, justify geometric relationships, and judge and defend the reasonableness of answers as applied to real-world situations. These students should be able to analyze and

interpret data in tabular and graphical form; understand and use elements of the function concept in symbolic, graphical, and tabular form; and make conjectures, defend ideas, and give supporting examples.

**Advanced** Twelfth-grade students performing at the Advanced level should consistently demonstrate the integration of procedural and conceptual knowledge and the synthesis of ideas in the five NAEP content strands.

Twelfth-grade students performing at the Advanced level should understand the function concept and be able to compare and apply the numeric, algebraic, and graphical properties of functions. They should apply their knowledge of algebra, geometry, and statistics to solve problems in more advanced areas of continuous and discrete mathematics.

They should be able to formulate generalizations and create models through probing examples and counterexamples. They should be able to communicate their mathematical reasoning through the clear, concise, and correct use of mathematical symbolism and logical thinking.

# Appendix B

## NAEP Mathematics Project Staff and Committees

# NAEP Mathematics Project Staff and Committees (1991–1992)

## Staff

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